90638

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2005

90638 Manipulate real and complex numbers, and solve equations

Credits: Five
9.30 am Wednesday 16 November 2005

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure that you have a copy of the Formulae and Tables booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–10 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**				
Achievement		**Achievement with Merit**		**Achievement with Excellence**	
Manipulate real and complex numbers, and solve equations.		Solve more complicated equations.		Solve problem(s) involving real or complex numbers.	
Overall Level of Performance					

You are advised to spend 40 minutes answering the questions in this booklet.

Show **ALL** working.

QUESTION ONE

(a) p and q are complex numbers, where $p = 5 + 7i$ and $q = 3 - 4i$.

Find $\bar{p} - 2q$, expressing your answer in the form $a + bi$.

(b) u and v are complex numbers, where $u = 4\text{cis}\dfrac{\pi}{6}$ and $v = 5\text{cis}\dfrac{\pi}{4}$.

Find uv, leaving your answer in the polar form, $r\,\text{cis}\,\theta$.

(c) Write $\dfrac{9 + 2\sqrt{3}}{5 - 4\sqrt{3}}$ as a number in the form $a + b\sqrt{3}$,

where a and b are rational numbers.

(d) Write $\left(2\operatorname{cis}\dfrac{\pi}{6}\right)^5$ as a complex number in rectangular form, $a + b\mathrm{i}$.

QUESTION TWO

(a) Solve $x^2 - 6x + 29 = 0$ expressing the solutions in their simplest form

$x = a \pm b\sqrt{c}\, i$, where *a, b, c* are rational numbers.

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(b) Fully factorise and hence solve $2x^3 - x^2 - 8x + 4 = 0$.

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(c)   Solve $2^{x-1} = 7$.

QUESTION THREE

Solve the following equation for x in terms of p:

$\ln(5x - 3) - \ln(x + 1) = \ln p.$

QUESTION FOUR

Solve $z^3 = -125$, where z is a complex number.

Leave your answers in polar form, $r \operatorname{cis}\theta$.

QUESTION FIVE

Solve the following equation for x in terms of k:

$$\sqrt{x - k} = \sqrt{x} - 2$$

QUESTION SIX

Find all the solutions of $(z^2 + 1)^4 = 1$, where z is a complex number.

Extra paper for continuation of answers if required.
Clearly number the question.

Question
number

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Question number